

Integer addition

Question:

How do you know that the answer to $-8 + 5$ is -3 ?

Possible answers to the question are:

1. The signs are different so you subtract and keep the sign of the larger number.
2. Imagine you have a debt of \$8. You find \$5. Now your overall financial state is a debt of \$3.
3. Suppose you are playing hopscotch on a number line. You are at -8 and take 5 steps to the right. You end up at -3 .
4. How can you add a negative to a positive? A negative says there is nothing there to add.

Response 1 is a procedural response. It does not provide an explanation for why the procedure works. Also, there is an error or lack of clarity in the response. Technically 5 is larger than -8 . Therefore the answer should be $+3$, not -3 .

Responses 2 and 3 concern an explanation for why the answer is correct. Response 2 makes use of a having/owing model. Response 3 makes use of a number line model.

Response 4 is not valid. It indicates a lack of understanding about what integers are and how they can be used to represent certain situations.

Note:

The notation system for integers (called flat notation here) used by engineers, scientists, technicians, etc. can confuse students if you begin the development of integer addition with it. The reason is that the symbols '+' and '-' can refer to an arithmetic operation (add or subtract) as well as to an integer (positive or negative).

For example, in ' $-8 + 5$ ', the '-' before the '8' means that 8 is negative and the '+' before the '5' signals the operation of addition. In ' $5 - 8$ ', the missing sign before the '5' signals that '5' is positive. The '-' before the '8' can signal that negative 8 is being added to 5 or it can signal that positive 8 is being subtracted from 5.

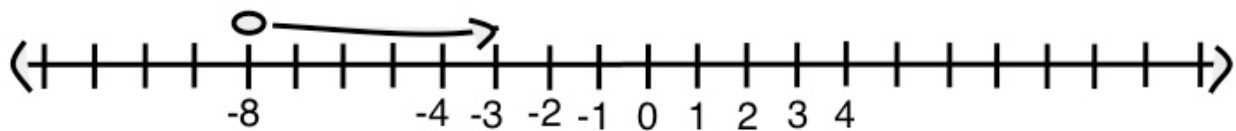
A teacher has to be aware of notation issues. The sample 3 stages lesson on integer addition begins by using bracket notation [e.g. $(-8) + (+5) = -3$] and later on in the lesson shifts to using flat notation (e.g. $-8 + 5 = -3$).

Models for teaching integer addition

A variety of teaching models can be used to develop integer addition. The most effective models are the following:

Number line model

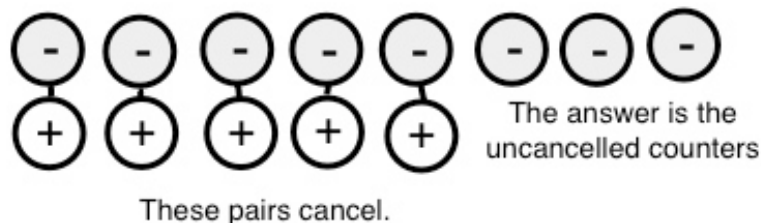
This model involves going for a walk on the number line. You start at the first integer and walk in the direction indicated by the second one. The answer is where you end up. For example, for $-8 + 5$, you start at -8 and move 5 steps to the right.



An issue with this model is, of course, the matter of questions such as $2 - 6$. The question needs to be interpreted as positive 2 ADD negative 6 so that the direction to walk in is clear and consistent in comparison to such questions as $-8 + 5$ (negative 8 ADD positive 5). **NOTE: It is important to interpret questions in an addition way. Using bracket notation makes it easy to see that the operation is addition.**

Electric charges model

This model involves using two-colored counters (e.g. red on one side and black on the other) to represent positive and negative integers. Addition is done by laying out the necessary number of counters and combining them. In that process, each pair of positive and negative counters (negative 1 add positive 1 = zero) is cancelled. For example, for $-8 + 5$, five pairs of positive & negative counters cancel, leaving 3 negative counters as the answer.



The main issue with this model is, as with the number line model, the matter of questions such as $2 - 6$. The question needs to be interpreted as positive 2 ADD negative 6 so that positive and negative are clear and that the question concerns addition, as indicated by combining counters. **NOTE: It is important to interpret questions in an addition way. Using bracket notation makes it easy to see that the operation is addition.**

Gain and loss model

This model involves contexts that concern gains and losses. Suitable contexts could be money gained and lost; increase and decrease in elevation; increase and decrease in scores; increase and decrease in temperature; and so on. Students use the context as a “natural” way to think about integer addition. For example, suppose the context is golf scores and the question is $-8 + 5$. The golf context would lead students to see the question as 8 below par combined with 5 above par, resulting in an overall score of 3 below par.

As before, one issue with this model is the matter of questions such as $2 - 6$. The question needs to be interpreted as positive 2 ADD negative 6 so that positive and negative are clear and that the question concerns addition, as indicated by combining counters. **NOTE: It is important to interpret questions in an addition way. Using bracket notation makes it easy to see that the operation is addition.**

A second issue with the gains and losses model is that the context **MUST BE** familiar to students. Otherwise, students will not be able to connect it to integer addition in a meaningful way.

Refer to: [Grade 7 Integer addition \(7.N.6\)](#) **if more help needed.**